

The Ancestry of 'Coherent Geometry' in Insular Designing

ROBERT D. STEVICK

Recognition of 'coherent geometry' in designing Insular art has spread from vernacular religious poems of the Anglo-Saxons, to the superb 'carpet pages' in the Kells and Lindisfarne Gospels (among others), to early high crosses of Ireland, and to the metalwork masterpieces of the Tara and Hunterston brooches. This paper undertakes to answer two questions about this element of Insular designing: What is its essence? and what is its ancestry?

In the past two decades, recognition of a distinctive element of design in Insular art has spread from vernacular religious poems of the Anglo-Saxons, to the superbly crafted 'carpet-pages' in the Kells and Lindisfarne Gospels (among others),¹ to early high crosses of Ireland,² and to the metalwork masterpieces of the Tara and Hunterston brooches.³ In design after design—no two quite the same—the proportional linkage of line with line, the proportional fit of every part with the whole, can be described in terms of linked ratios of measure, and it can be replicated by a sequence of simple, continuous manoeuvres of compass and straight-edge. 'Coherent geometry' is the term selected here for this organic aspect of designs. It was coined to characterise the full binding of a formal plan by linked proportional relations among all its parts, those relations accordant to ratios generated in the most elementary constructional geometry. 'Accumulated geometry' would be equally appropriate, being a term devised to imply the procedure by which that complete proportional binding is brought into being; the term is a calque on J. S. Bach's understanding of harmony in composition as accumulated counterpoint. Or it could be called 'harmony of disciplined proportion', to foreground harmony, identify its material, and specify discipline as an essential in its creation.

It is remarkable that a method or principle of designing (whatever we choose to call it) should be employed in art forms as diverse as vernacular poetry, manuscript illumination, stone sculpture, and ornamental metalwork. Remarkable, too, is that this apparent aesthetic imperative operated over such a long stretch of history.

For both these reasons, this element of Insular design asks for double accounting: what is its essence? and what is its ancestry? Inappropriate, I think, will be any attempt to theorise this principle, or any attempt to historicise it. Instead, I think it will be helpful to review briefly some of its always-present traits, and then to introduce some of its ostensible antecedents.

Box 354330, University of Washington,
Seattle 98195,
USA.

JRSAI Vol. 134 (2004): 5-32

E Sep 2006

ITS ESSENCE

As with any principle of form in the fine arts, iteration is a key feature. With Insular design, this repetition of a motif or trait is of a kind fundamentally different from the kind which informs stanzas for verse composition, or fretwork twilled or trellised, or step and key and grille patterns used in decoration. With these the engagement of attention is to confirm the regularity of the line-groupings, for stanzas; or, for ornament, to confirm each band or thread or line having perfect and singular continuity: every element that enters anything knot-like emerges intact. Any continuity or engagement is merely repetitive, however. It makes us ask, Does it do every time what I traced in the first instance? If the device is well done, the repetition is perfect. The visual-tactile test is carried out, and its conclusion squares with the immediate grasp. There is an exercise in tracing the elements of the design, and retracing and retracing, but without development of the idea.

Nearer in kind to coherent geometry is the interlace frequently found in the Insular tradition, characteristically used for filling areas. Although it is a single-surface pattern, it seems to have depth: it appears immediately as a weaving or lacing in which an end of a band or thread has gone under and over another band or thread, and does this again and again. But follow the band, whence it came and whither it goeth. It then becomes obvious through a tactile testing of the plan that the thread could not have gone under and over another one because, in the best examples, at least, there is no end and in some cases there is no other band. There need be only one continuous band or thread forming the entire interlace pattern. (If more than one band, each one has no ends.) Here is a paradox: a band has no ends, yet it represents a lacing process which could be achieved only by pulling an end under and over other segments of the band. In this kind of design the immediate understanding is contradicted by the reflective, analytic, tactile-tested grasp. We comprehend in both modes, alternately and simultaneously, it seems.

The Möbius strip—illustrated unforgettably by M. C. Escher—while not found in Insular art, illustrates this principle even more clearly. The strip has no ends, and in that respect is similar to a circle, an ellipse, and whatnot. But it is a strip, having ostensibly two sides—two surfaces—as a circle does not. Then follow any surface and see where it is going, where it came from, and it becomes obvious that the strip has only one side. There is another paradox: the strip has two surfaces, except that when you move along it there is only one. (The same thing is true for the edges: there are two, as you see immediately, although if you trace the whole figure it becomes clear that there is only one.) The immediate grasp of the visual form—the subito perception—is in contradiction to the analytic or quasi-tactile grasp. Yet we comprehend in both ways. We confront a paradox without disbelieving either of its contradictory terms, but the idea develops no further.

Iteration in Insular coherent geometry is unlike the undeveloped repetition of stanzas, or of fret, step, or key patterns, and unlike the iteration of an endless loop in good interlace and Möbius strips. The iteration does not lie in shapes, or weaving sequences, or other methods producing the same form again. Rather, it is iteration of ratio—a relational fac-

tor among structural dimensions—that informs the disposition of all parts within a plan. Further, the relations among the parts regularly involve a counterpoint. Not a counterpoint of contradictory appearances, as with a Möbius strip or interlace. Instead, a counterpoint of perception of measures represented by rational and irrational numbers, in modern terminology. Or a counterpoint of arithmetical and geometrical proportions (as they have been called), in constructional terms.

The method of designing for coherent geometry, to be illustrated presently, is typically to set a given measure, then using elementary constructive geometry divide it equally by two (division by a rational number), and then divide one of the equal measures into two unequal segments whose measures cannot be expressed in integer fractions (division by an irrational number); virtually without exception the latter division involves $\sqrt{2}$, $\sqrt{3}$, or $\sqrt{5}$. And then proceed to constructing the form of the cross, the page illumination, the poem, the disc, the brooch by constantly feeding back into the plan the measures that represent both rational and irrational numbers participating in the same design. The idea of iteration of this kind becomes open to endless lines of development.

The essence of coherent geometry in Insular design will be illustrated with a late (twelfth century) high cross. The ancestry will be illustrated with five disparate objects:

two La Tène bronze discs (fifth/fourth centuries B. C.), a bronze mount and a bronze disc from Ireland (first/second centuries A. D.), and a slate motif-piece from western Scotland (probably seventh century A. D.). The order of their discussion is not chronological, because no earlier one can be claimed as a source for another, any more than any later one should be thought of as 'reminiscent' of another. Instead, the order of presentation is based on typological similarities, to make it easier to recognise the sharing of common formal characteristics among these objects that distinguish this kind of design.

Kilfenora high cross The west cross at Kilfenora, Co. Clare (Fig. 1), stands 4.6 m in height, the cross-arms spanning 1.3 m; it is dated to the twelfth century.⁴ This cross is typical in nearly every way of the design of



Fig. 1 West Cross at Kilfenora, Co. Clare. Photo: Department of the Environment, Heritage and Local Government.

Irish high crosses, including its accumulated geometry. It is fully typical in its symmetries, less so in its aberrations from symmetry. As will appear, though, the coherence of the plan of this cross is manifest in both its symmetries and its departures from them.

There is conspicuous distortion of symmetry in the arms of the cross. Their ends slant inwards from the lower to the upper limits, and their thickness (i.e., vertical measure) is much greater above the mid-line of the cross than below it. That leaves the quadrant re-entry arcs within the ring (the 'armpits') unbalanced, because those arcs must terminate at inconsistent positions within the ring. There is further imbalance in the plan of the upright portion of the encircled cross: the width (horizontal measure) of the upper element is somewhat less than the width where the lower element intersects the ring; the effect is to splay the shaft—i.e., to spread the re-entrant arcs in the lower half of the ringed portion of the cross. The imbalance of the re-entrant arcs then is continued in the imbalance in placement of the half-cylinder devices ('volutes' or 'rolls') ornamenting the inner surface of the ring. They are not aligned on axes diagonal to the rectangular plan of the cross, having been placed instead (roughly) midway within the re-entrant arcs. The misalignment is obvious in relation to the diagonal straight-line ornament within the central circle of the plan.

What remains of the elementary geometry of the basic design, still clear despite these distortions and imbalances? The underlying plan is very simple to plot, and very much like that of other sculptured crosses of its time and place.

First, an operational account—a how-to narrative—to go with diagrams (Fig. 2) explaining how to replicate the cross's form. The sequence of drawings is indicated by alphabetic marking (2a, 2b ...). Within each drawing the sequence of construction steps is identified with numerals (2a.1, 2a.2 ...). In this account and in the others to follow, 'copy' is the most common directive. It should be carried out with compass (dividers) and not with a measuring scale: set the points of the instrument at the ends of the length to be copied, then move one of these points, as directed, to mark the same length at another place in the plan. 'Double' is done in the same way, copying a measure once along a straight line.

- 2a.1 Describe a circle *A*.
- 2a.2 Next, run a straight line—a diameter—through the centrepoint of the circle, dividing the circle into halves.
- 2a.3 Set a second diameter perpendicular to the first one, dividing the circle into quadrants.
- 2a.4 Copy the radius length of Circle *A* from each end of the axes to set intersecting arcs outside the circle; lines connecting opposite pairs of intersections would set third and fourth diameters dividing the quadrants equally (cf. Fig. 2e).
- 2b.1 Copy the measure of the chord of a quadrant arc to mark a point along a diameter, and using the distance from there to the centre as radius, describe a new circle *B* concentric with circle *A*.

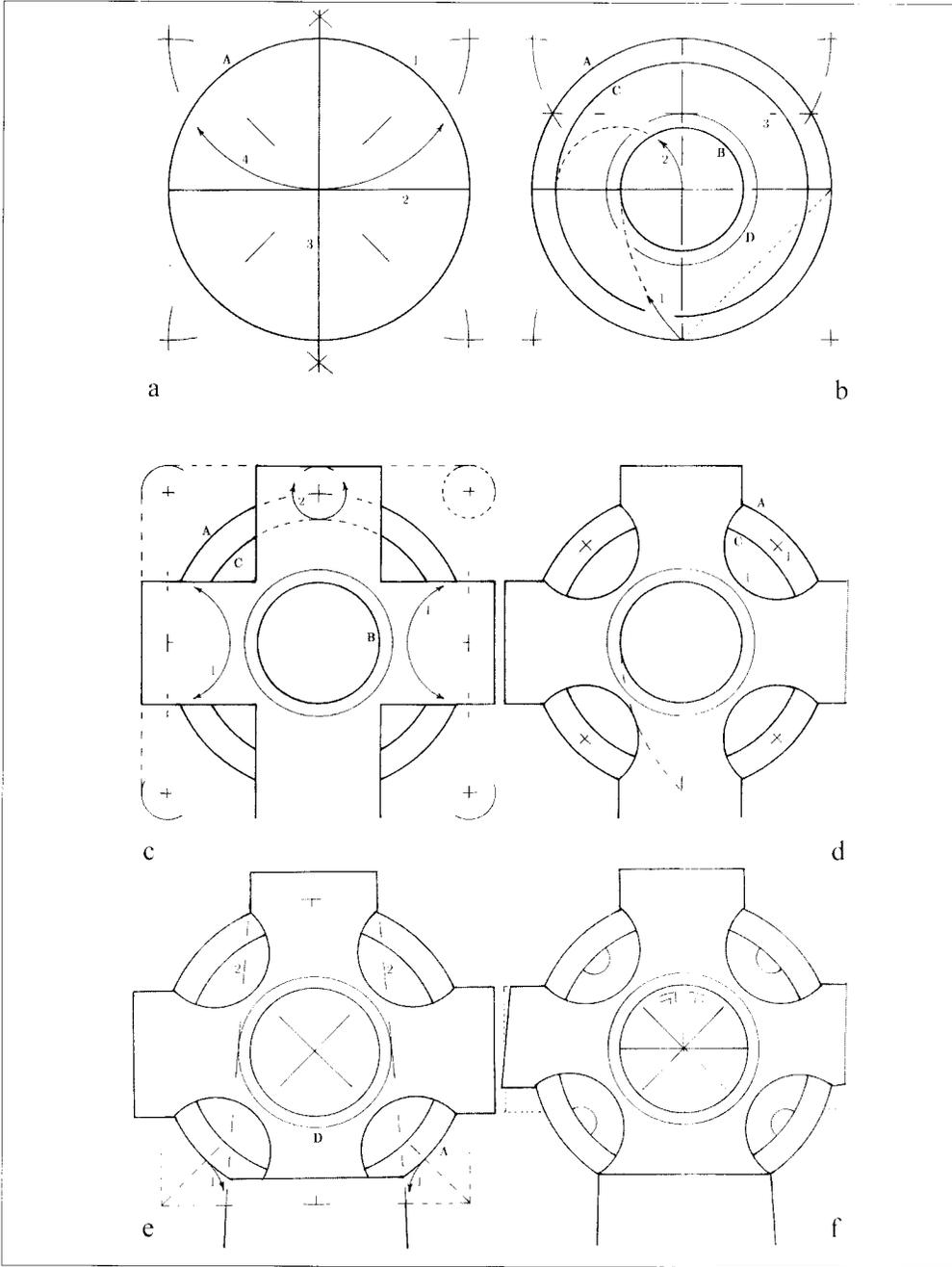


Fig. 2 A way to design the Kilfenora Cross.

- 2b.2 Double the radius of circle *B* for a new circle *C* concentric with the two others.
- 2b.3 Halve the radius of circle *A* for a new circle *D* concentric with the three others.
- 2c.1 From each end of the first two diameters, copy the radius length of circle *B* to mark points on either side along the sides of the square commensurate with circle *A* (corners as set in 2a.1). Connect these points of intersection in pairs with parallel lines to set the preliminary shape of the cross.⁵
- 2c.2 Plot the extensions of the arms and the upper portion of the shaft of the cross beyond circle *A* by as much as circle *A* exceeds circle *C* in radial measure. (This is another 'copy' operation.)
- 2d.1 Find the midpoint between circles *C* and *A* along the diagonal diameters; one way is to mark the combined (i.e., end-to-end) radius lengths of circles *B* and *D* along these diagonals.⁶ Using these points as centres, describe arcs extending to the intersections of the preliminary cross and the outer circle *A*, in order to form the provisional re-entrant arcs for this cross design.

The design in this construction accurately matches the shape of the cross (west face) wherever the stonework is symmetrical on both axes.

- The central circle *B*—the one enclosed by the rope-like circle in the sculpture— exactly matches the model, having exactly half the measure of the inner circle of the ring *C*.
- Circle *D*—the outer limit of the rope-like carving—has a diameter one-half that of circle *A* when measured along the middle diagonal lines of the central decoration.
- The top (vertical) extension of the cross joins the ring (circle *A*) with the same lateral measure as the central circle (*B*).
- The height of this top extension agrees with the model.
- The extensions of the cross arms, along their upper lines, would if joined be tangent to the central circle *B*.

That is where the full symmetry of the sculpture ceases and the point at which the first stage of the plan and the artefact diverge.

Before trying to grasp the cause or source of the asymmetry, it should be observed that decoration of the shaft on which the ringed cross rests has its own formal scheme consonant with that of the key element that it supports. We cannot know whether the tall supporting pillar was designed by the same person who set out the design for the circle-and-cross that sits above it. Whoever designed it, however, must have understood the design of the ringed cross it was created to hold up, as the decorative scheme makes clear. The long rectangle enclosing assorted ornament in interlace, on the reverse, 'entièrement recouvert d'ornements finement gravés dan la surface de la pierre.'⁷ represents yet another 2:1 relation, its height being twice the diameter measure of the ring enclosing the cross. More than that, coming down the rectangle are double layers of interlace patterns. The first

two fill one-fourth of the vertical measure. The next two fill another one-fourth. Below that begins another pair, before the decorative scheme was left incomplete.

And yet, between the measure and half-measure of the ring (its diameter and radius) and the layered measures of the supporting pillar there is no meld: the top of the pillar intrudes into the ring to about the path of its inner circle. This gives added stability to the join of shaft and ring-and-cross, but there is no practical need for that. There may be aesthetic need for it, of course, if the shaft is to be both long and tapered: the path of the tapering leads to the upper extension of the cross on one side, though it misses slightly on the other. In any case, while the modules for the ring design and the pillar decoration are the same, it is not continuous between the two elements of the cross, and its uses in the two parts are not linked in any apparent way.

Rather than reason from putative aesthetic requirements, though, it may be better to pursue the geometrical underpinning of the plan developed first in full symmetry, and see whether the asymmetry of the upper and lower halves may be part of that plan. It will turn out that the spread of the cross-shaft within the ring follows a simple and continuous derivation of dimensions consistent with the rest of the plan.

- 2e.1 From each lower corner of the large underlying square (corners set in 2a), copy the measure from there to circle *A* to mark points along the lower side of the square. (These points can also be set as the measure from circle *A* to circle *B* copied on either side of the vertical axis.)
- 2e.2 Then on either side, sketch a line between one of these points and the corresponding top corner of the cross-shaft (each will be tangent to circle *D*). Where these lines intersect circle *A* will mark the top corners of the supporting pillar and its join with the encircled portion of the cross.
- 2f The top corners of the pillar also mark the lower ends of the re-entrant arcs in the lower half of the plan. To keep the compass setting of the preliminary plan constant for these 'armpits' requires moving the centres of the arcs upwards around the ring; that in turn moves the upper ends of these arcs upwards, which in turn moves the lower edges of the cross-arms upward. (The upper edges remain as originally plotted.)

What occurred in the process of designing the Kilfenora west cross may have been something like this. First came decision on the scale of the cross and its style. In this instance the scale was to be quite large, at least for its height, with a tall supporting pillar to stand as a long lower shaft of the cross. The style was to include a tapered shaft.⁸

The design was to proceed in full conformity with the traditions of design for sculptured crosses, which had been used as well for page illuminations in Gospels codices and for the finest ornamental metalwork—that is, it would evolve as an accumulation of relationally coherent derivations that are carried out with compass and straight-edge. It must have been undertaken in the manner of the derivation of what above was called the pre-

liminary plan (2a-d). Every aspect of the plan at this stage was linked to all others by a tactile procedure that produced simple proportional relations among all the lines shaping a fully symmetrical ringed cross.

Then came the necessity for decisions on melding the ringed cross and its tall and tapered supporting column. At this point there was a critical choice in the creation of the design: whether to make the ringed portion symmetrical inversely as well as laterally, to be set atop a tapered extension of the cross's shaft? Or to introduce taper into the ringed part of the sculpture? The choice was for the latter of these options, possibly as an attempt to introduce a primitive form of perspective, enhancing the perception of height for a cross-head atop a very tall pillar.

And with that choice came another, how to set the slant for the taper? The designer chose to continue the mathematical regimen of his underlying plan. The measure by which the horizontal portions of the cross exceeded the radius of the ring would also be the measure by which the lowest part of the shaft would exceed its originally plotted breadth.

At this point it may be useful to itemise some of the coherence of the plan's proportional structure, beginning with repeating elements. The relation between the sizes of circles A and C , comprising the ring, is the same as the relation between the diameters of circles D and B . These are the pairs of concentric circles of the plan that form the rings. That is a verbal rendering of the equivalent ratios. Recognising how these came to be equivalent is another way to grasp the equivalence. Circle D was derived as having half the measure of circle A , while circle C was derived as having twice the measure of circle B ; each pair is related as 2:1. Expressed yet another way, $A/D = C/B$. That being the case, it will also be the case that $A/C = D/B$.

And what specifically is the relation of A to C , and of D to B ? Again by attending to the constructional process, it is clear that as much as the chord of a quadrant exceeds the radial measure of a quadrant is the measure of the radius of the inner circle B . See Fig. 2b.2, lower quadrants, where a right triangle can be recognised with radii of circle A for its sides, the chord of a quarter-circle forming its hypotenuse. Let the radius of circle $A = 1$. By the Pythagorean theorem, the chord (hypotenuse) $= \sqrt{2}$. In modern notation, then, the radius of circle B is $\sqrt{2} - 1$ in relation to the radius of circle A . Now, circle C has twice the radius of circle B , which is to say, $2(\sqrt{2} - 1)$. In relation to the radius of circle A , then, the ratio between circles C and A is $2(\sqrt{2} - 1):1$ (the decimal approximation is 0.8284.) The relation of circles D and B , as noted, is the same.

In terms of constructive geometry there are other points of interest. Recombination of the basic measures (as noted earlier) will set the centres of the re-entrant arcs at a distance from the centre equal to the sum of radii of circles B and D . The measure copied in 2e.1 is identical (in length, not in derivation) to the measure used for the radius of circle B , and re-used for arm and upright thicknesses at the outset. The re-entrant arcs set in 2d.1 also are tangent to the path of the compass in 2b.1, used to set the radius of circle B , and subsequent measures. And so on.

In sum, the whole design of this cross—apart from its surface decoration—proceeds in an unbroken, disciplined chain of geometrical derivations of a most basic kind. Taken together, the imbalances and distortions, along with the symmetries of this cross, offer an unusually instructive example of high cross design, for its concepts and pragmatics together. Many other crosses have designs fully answerable to schemes of coherent geometrical construction, but in less complex ways, for being mainly symmetrical throughout.⁹

The creator of this cross plan proceeded in the customary way in laying out a ringed cross, to be enhanced by its re-entrant arcs. He then proceeded to introduce the tapering of the cross shaft within the encircled portion of the cross. He held to the regimen of geometrical coherence in setting the taper. That done, he had to accommodate the re-entrant arcs to the unparallel lines of the cross shaft. And that entailed moving the cylinder-like devices inside the ring away from their true diagonal positions. By not departing from the mathematical regimen in drawing the re-entrant arcs, he distorted the cross-arms along with the diagonal symmetry of the re-entrant arcs. There was no way of eliminating the distortion at this stage, and it could be reduced only by ad hoc adjustments—by fudging the disposition of the re-entrant arcs. The designer did not do this, although he altered the ends of the cross-arms by slanting them approximately parallel to the taper of the upright, perhaps as further attempt to enhance the sense of loftiness of the cross-head within the ring.

In its thoroughgoing commitment to the rules of its tradition, even when that entails ostensibly misshapen cross-arms and other asymmetries, the Kilfenora cross embodies the essence of coherent geometry in Insular design.

ITS ANCESTRY

Cuperly disc The same method, the same rules, the same discipline in devising harmonious proportion—even the same ratio for the circles of its ring—inform the design of the Cuperly disc, Marne (Fig. 3). This bronze piece, measuring 10.8 cm in diameter,¹⁰ was designed more than a millenium and a half earlier than the Kilfenora Cross, in a culture having clear continuity with that of early Irish art.¹¹ Without asymmetries like those in the Kilfenora Cross, a derivation of its design is simpler to describe.

Construction of its form (Fig. 4) begins with exactly the same steps employed in deriving the Kilfenora Cross form.

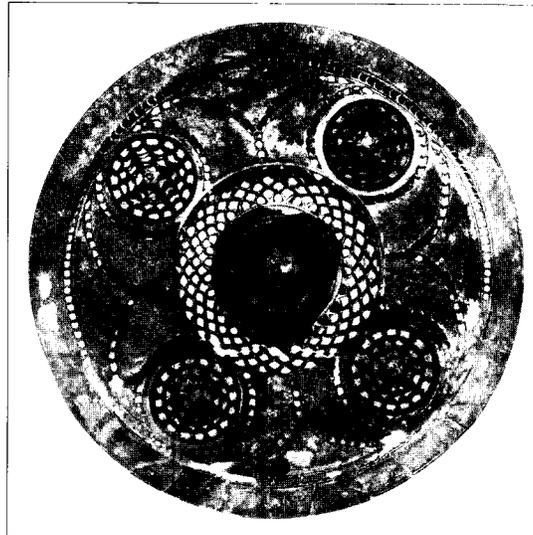


Fig. 3 Bronze openwork phalera from Cuperly (Marne), M.A N. 27719. Photo: Musée des Antiquités de Saint-Germain-en-Laye.

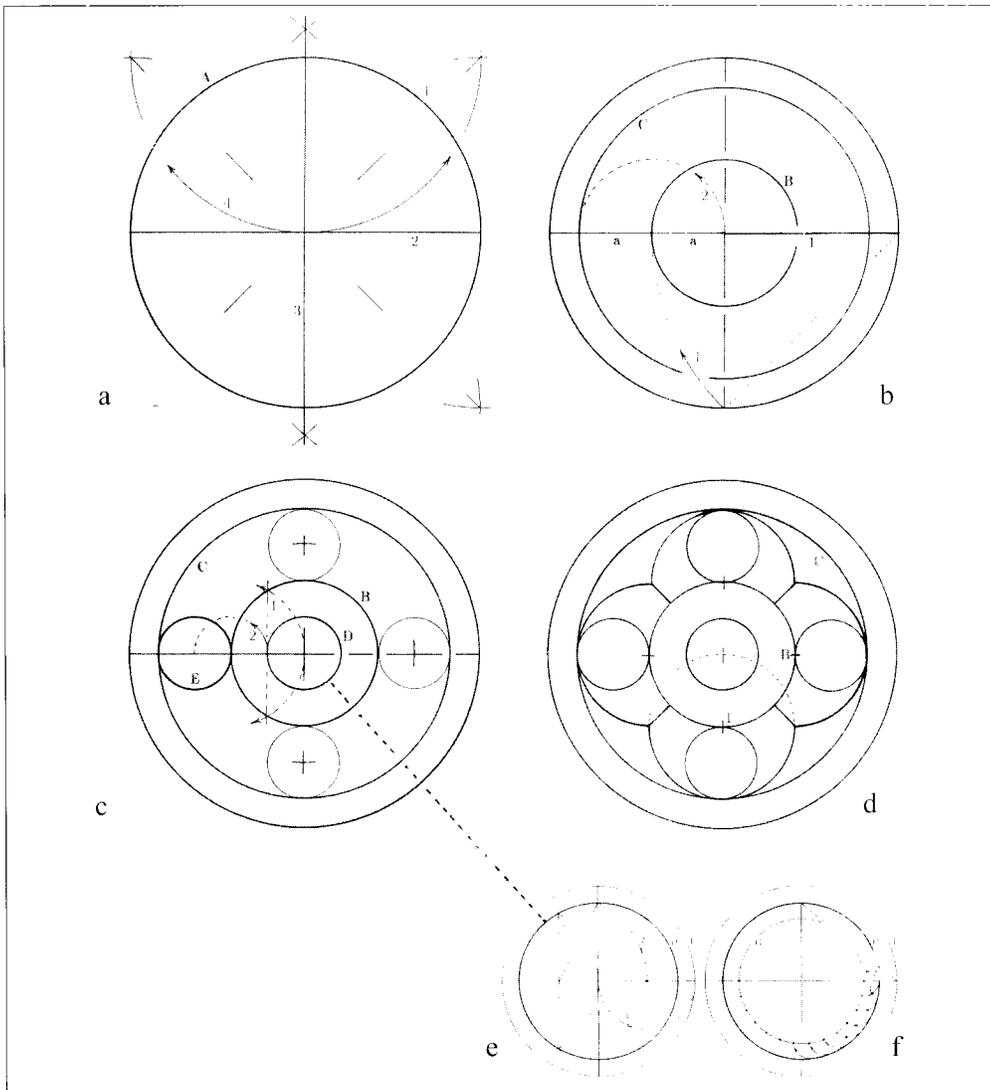


Fig. 4 A way to design the Cuperly disc.

4a.1 Describe a circle A .

4a.2 Run a straight line through the centrepoint of the circle, dividing the circle into halves.

4a.3 Set a second diameter perpendicular to the first one, dividing the circle into quadrants.

- 4a.4 Set third and fourth diameters dividing the quadrants equally, along diagonals of an underlying square commensurate with circle *A*.
- 4b.1 Copy the measure of the chord of a quadrant arc to mark a point along a diameter, and describe a new circle *B* concentric to the original one, with radius to the measure just marked.
- 4b.2 Double the radius of circle *B* for a new circle *C* concentric with the two others.

At this point the derivations of stone cross and bronze disc will diverge.

- 4c.1 Halve the radius of circle *B*, and with this new measure as radius describe circle *D* concentric with the three others.
- 4c.2 Copy that last radial measure beyond circle *B* along a diameter, and describe circle *E* with that same radial measure, tangent to circles *B* and *C*; repeat circle *E* along the vertical and horizontal axes.
- 4d.1 Use the intersections of circle *B* and the horizontal and vertical axes as centres for circles both tangent to circle *C* and passing through the centre of the plan; segments are used in the Y-shape pattern in each quadrant. (Alternatively, the circles could have been sketched in carrying out the operations in 4b.)

For the Cuperly disc, as for the Kilfenora Cross, the initial circle (4a.1) sets a given measure as 1 (the radius) and doubles it as 2 (the diameter). The next measure to be set (4b.1) divides the radius unequally, as explained earlier. To display the coherence of the plan in a different mode, let us designate the shorter segment *a*, which is the 'geometrical' irrational quantity $\sqrt{2}-1$, as explained above. It can be said then that ratios combining the three quantities 1, 2, *a* inform the complete layout of both pieces. They provide a three-member alphabet, as it were, in which can be written the code of this design and would be equally appropriate to that of the Kilfenora Cross. Some of the coherence of this plan's proportional structure shows clearly in Table 1, especially in the column giving the basic dimensions in terms of the primitive quantities 1, 2, *a*.

Table 1 The Cuperly disc.

RADIAL MEASURE	IF <i>a</i> STANDS FOR $\sqrt{2}-1$	APPROXIMATION
Circle A = 1	1	1.0000
Circle B = $\sqrt{2}-1$	<i>a</i>	0.4142
Circle C = $2(\sqrt{2}-1)$	2 <i>a</i>	0.8284
Circles D, E = $(\sqrt{2}-1)/2$	$\frac{1}{2}a$	0.2071

Beyond the basic plan, the Kilfenora Cross had its taper; the Cuperly disc has its open-work ring surrounding the central disc generated from crossing patterns of twenty-six

arcs. The inner limit of these arcs is circle *F*, the outer limit circle *B*. The derivation can be carried out as follows.

- 4e.1 Divide a horizontal radius of circle *D* in half.
- 4e.2 Copy along the diameter the hypotenuse of a triangle with sides equal to the radius and the half-measure of the radius, from the half-measure of the radius opposite.
- 4e.3 Double the longer segment of the unequally divided radius to set a new radius measure for circle *F*.¹²
- 4f.1 Sketch the hypotenuse of a triangle with sides equal to the radii of circles *D* and *F*.
- 4f.2 From the outer end of the horizontal radius of circle *D*, copy the vertical (perpendicular) measure to that hypotenuse along the radius inside circle *D* to mark the radius measure for circle *G*.
- 4f.3 Intervals between the centres of the intercrossing arcs are copies of the distance between circles *G* and *D*.

The radius of the fretwork arcs is equal to the diameter of circle *G*; the fretwork itself consists of those arcs traced between circles *D* and *B*. The result, in terms of coherence of the geometry, is this:

Circle *F* has a radius measure $\sqrt{5}-1$ that of circle *D*.

Circle *G* has a radius measure $1/(\sqrt{5}-1)$ of circle *D*.

Put another way, circle *F* exceeds circle *D* in the same ratio that circle *D* exceeds circle *G*—symmetrical ratios (not symmetrical measures).

The foregoing description provides new information about the design that was not included (or attempted) in earlier descriptions—specifically, the procedures of construction, from a single given measure, that yield fully and accurately the overall form of this disc and the design of the open fretwork as well.¹³ This ‘as constructed’ account for the first time shows the coherence of the whole design in terms of its geometrical proportions. The basic form (4a-d) is created from relations among 1, 2, and $\sqrt{2}$, while the openwork arcs and intervals (4e) are created from relations implicit in ϕ (‘golden ratio’ i.e., $2/(\sqrt{5}-1)$); the radius of circle *F* is $2/\phi$, and the radius of circle *G* is $\phi/2$. At the same time, it in no way contradicts the very important analysis published by Frey and Schwappach,¹⁴ with its analytic diagram often reproduced in subsequent publications. Their interest was in the use of compass to construct figural motifs of ornament, and to characterise styles of ornament and their distributions geographically and over time. ‘On the Cuperly disc four circles with “blossoms” in between them are arranged around a circular centre.’ etc. Other observations such as that of L. and J. Laing, that the plan is an arrangement of five roundels, have been on the same order.¹⁵

How do these first two pieces compare generally? The disc has a diameter of 110 mm, while the Kilfenora Cross is 1.3 m in breadth. One is bronze, one is carved stone. One may be only secular and ornamental, the other religious and symbolic. Their dates and provenance are widely separated, as noted earlier. Size, material, purpose will be irrelevant, however, to understanding both the typological and the genetic relations of the two designs. Their respective venues, on the other hand, at least predict that they may be related.

It is finding of the same method, the same rules, even the same informing ratio—these shared features of cross and disc designs—that makes as strong an argument for genetic relation as does any string of similarities in motifs of decoration typically noted in Insular art studies.

The typological relations between the two basic configurations are not recondite. Within a continuing tradition of creating designs by accumulated geometry, only two variant developments would be needed. One is superimposing a cross pattern on the circular plan: all it would take in this instance is copying one dimension—the radius measure of the 'geometric' inner circle—at the ends of the horizontal and vertical axes, setting the basic dimensions for the arms and the shaft of the cross. This development, as in 2c of the Kilfenora derivation, produces a plan that resembles the very early St Kevin's Cross, Glendalough, Co. Wicklow, for example. The other is turning inside-out (so to speak) the development of the design along the diagonal axes: move the centres of arcs from the horizontal and vertical axes to the diagonals of the design and from the inner circle (*B* in these examples) to the outer ring. This, as in 2d, aligns the re-entry arcs ('armpits') that became a dominant feature of Irish high cross design. The clear implication, I believe, is that the later design is in fact merely a variant realisation of the underlying principles of design that had shaped the Cuperly disc.

Dunadd motif-piece Further items in the ancestry of Insular designing can be described more economically. The partial plan of the Dunadd motif-piece (Fig. 5) will illustrate 'coherent geometry' in yet another way. It looks like a scruffy bit of work—small, sketchy, criss-crossed with spurious markings; the diameter of the sketched form is about 36 mm, and the piece dates probably from the seventh century.¹⁶ Parts of it are compass-drawn, parts are drawn free-hand. In the orientation of a brooch—which the piece prefigures—the upper portion has concentric arcs within the circle, one full, one partial, with their shared centre clearly visible, all drawn by

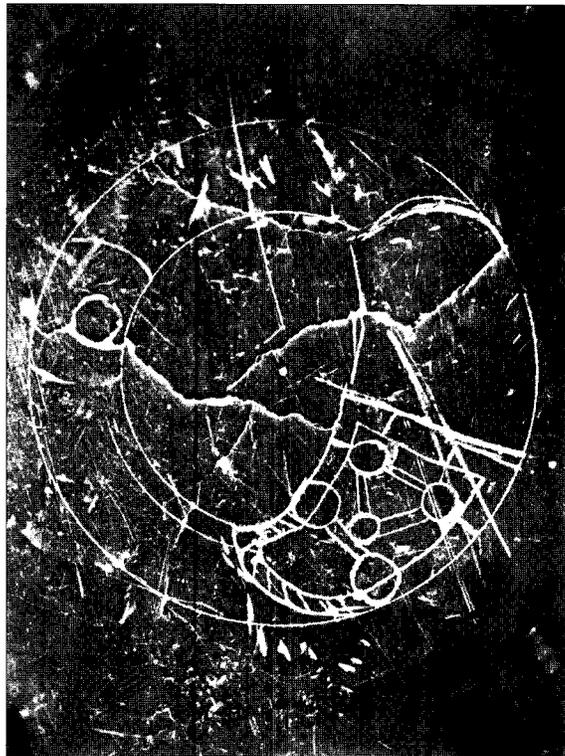


Fig. 5
The Dunadd motif-piece. Photo: Dr Niamh Whitfield.

compass. In the lower portion, one arc appears to be at least partially compass-drawn, the broad arc at the top of the lower portion. A smaller arc on the left-hand side seems to be compass-drawn, with free-hand overdrawing. All the rest in the lower half—circles, curves, straight lines—appear to be free-hand drawings. The free-hand portion of the plan shows the left-hand side of a tentative scheme for decoration of the lower segment of the brooch, the disposition of cells for precious stones, or interlace, or filigree, or whatever. (No need to sketch the right-hand side, which would be merely the same pattern in reverse.) This part of the design is little more than an inventory of a decorative scheme yet to be designed for this part of the brooch's face. As roughly drawn, it contains almost no information about the coherent geometry of this part of the design. As the design may have developed, of course, the elements in this area could well have been plotted according to the geometrical scheme that is present in the compass-drawn portion.

The coherence of the plan evolves like this (Fig. 6):

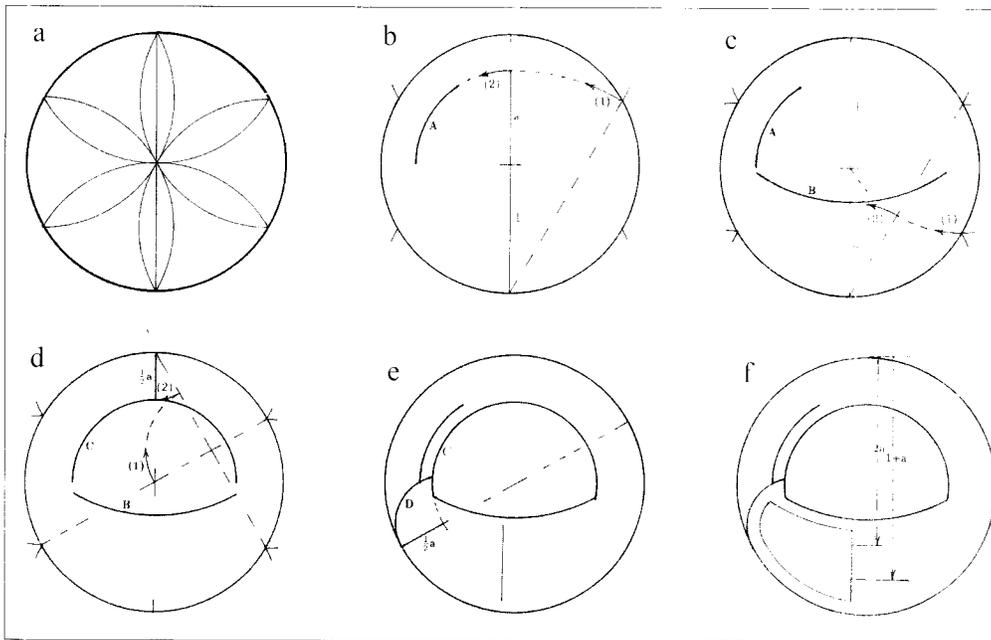


Fig. 6 A way to replicate the design in the Dunadd motif-piece.

- 6a Begin by drawing a circle. To set the bilateral symmetry of the plan, run a vertical line through the centre of the circle (a diameter). Development of the plan will further require division of the circumference by six—that is, each half of the circle will be divided equally in thirds. To use a perhaps familiar example, one way is to keep the

- compass set with the radius measure of the circle and proceed to mark its length successively around the circumference. If the swing of the compass is traced, the daisy-pattern in the drawing is the result. But for this plan, simply mark the radius length along the outside of the circle, starting from either end of the diameter, as in 6b (next).
- 6b Call the measure of the radius 1. Then copy the measure between two divisions of the circumference (what would be the chord of $1/3$ of the circle) to mark a point along the vertical diameter, starting from the bottom of the figure. Finally, draw arc *A* concentric with the original circle, radius extending to the point just marked. Call the radial measure of the arc *a*.
- 6c Arc *B* just below the middle of the plan has its centre at the top of the circle, and a radius with length $2-a$. Its derivation can be either by copying measure *a* from the bottom of the plan, or by a fresh derivation such as the one shown in the figure. The distance between two adjacent divisions of the circumference is $1+a$; from that is (1) subtracted the measure 1, and the remnant measure *a* is copied (2) along the centerline. Or for that matter, it can be set by copying the measure 1 below the path of Arc *A* along the centreline.
- 6d The inner arc of the 'hoop' in this plan has a radius $1-1/2a$; in constructional terms, its radius is the difference between the circle's radius (=1) and half the radius of arc *A*, same as half the nearest measure between arc *B* and the circle. Any convenient method of halving a measure will be appropriate; the procedure shown uses a line between two adjacent divisions of the circumference—again a chord of one-third the circumference—to divide a radius of the circle equally in two. In turn, the radius divides the longer line, $1+a$, equally in two, each half measuring $1/2 + a/2$. Then the $1/2$ measure is subtracted from this length (1), leaving $a/2$, which is copied (2) along the vertical axis, setting the length of the radius of arc *C*. Essentially, this is the same method that produced arc *B*, but at half scale.
- 6e Arc *D* may be compass-drawn and then over-drawn by hand. In any case, where the path of arc *C* intersects a diameter running between the 8 o'clock and 2 o'clock positions is the centre of arc *D* (12 o'clock being at the top of the vertical diameter), and its radius is the same measure $a/2$ already established.
- 6f Two hand-drawn lines may participate in this plan as well—the roughly horizontal lines spacing the decorative cell from the central axis in the lower portion of the plan. The upper one seems to be $2a$ from the top, the lower one $1+a$ from the top (a measure that could be copied from 2, above).

The diameter of the compass-drawn circle on the piece measures 36 mm. A pen-and-paper model constructed on the same scale may not be persuasive of the method of designing proposed here: there could be too much margin for error, it would seem, given the tools and the medium of the slate sketch, not to mention possible limitations from imperfect eyesight and unsteadiness of hand. This analysis, therefore, was developed in a model on a much larger scale—more than four times the diameter of the original. And to keep the

model to a high degree of precision, all the constructional lines were checked for accuracy by calculating where they should be if the geometrical relations were being drawn precisely. (This is a procedure used in all the analyses for this paper and others I have offered.) On this large model was superimposed an enlarged photograph of the piece. The closeness of the match of model and artefact was astonishing.

The radial measures in this plan are shown in Table 2, the middle column again showing the sufficiency of three primitive geometrically related quantities to capture its geometrical coherence. To recognise the length called $\sqrt{3}$ in the first column, refer to step 6b. A right triangle occurs within any semicircle, in this case with the diameter (=2) forming the hypotenuse, one side of the triangle being the radius measure (=1). By the Pythagorean theorem, the other side is then $\sqrt{3}$.

Table 2 The Dunadd motif-piece.

RADIAL MEASURE	IF a STANDS FOR $\sqrt{3}-1$	APPROXIMATION
Outer Circle = 1	1	1.0000
Arc A = $\sqrt{3}-1$	a	0.7321
Arc B = $2-(\sqrt{3}-1)$	2-a	1.2679
Arc C = $1-1/2(\sqrt{3}-1)$	$1-1/2a$	0.6340
Arc D = $1/2(\sqrt{3}-1)$	$1/2a$	0.3660

Using modern notation (as in column 1 of the table), relations among the radii of the circle and the arcs link like this:

$$A/D = 2$$

$$B/C = 2$$

$$C/D = \sqrt{3}$$

$$B/A = \sqrt{3}$$

$$B/D = 2\sqrt{3}$$

$$C/A = (1/2)\sqrt{3}$$

One can set aside this modern notation, though, with all the measures and the coherent scheme of relations among them expressed just as truly and simply and comprehensibly in the tactile procedures that will produce the primary, compass-drawn portions of the plan of the Dunadd motif-piece, the procedures illustrated in Fig. 6. In any case, the piece seems to illustrate the trial-and-error process that must have occurred in any designing process in this tradition of art, in alternative arcs to set the thickness of the hoop, on the one hand, or in the free-hand markings, on the other, presumably to be guides to further choices among dimensions that were then to be 'rationalised'—i.e., plotted to conform to the scheme of ratios selected to govern the whole design.

Cornalaragh mount The next example of coherent geometry in Insular art is a piece located historically less than halfway between the Cuperly disc and the Kilfenora Cross: a mount, from Cornalaragh, Co. Monaghan (Fig. 7). Dated to the first/second century A. D., the piece 'was designed with a compass and then hand chiselled; diameter 7.4 cm'.¹⁷ The basic plan is again simple, as will appear. Surrounding its central circle is elaborate cut-out work similar to that of the Cuperly disc.

For this piece, let the drawings (Fig. 8) make up an ostensive description of the design. To that simple 'show' can be added the conventional 'tell'.

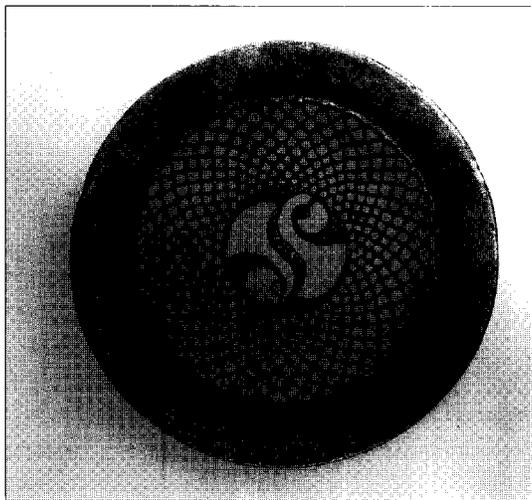


Fig. 7 Mount from Cornalaragh, Co. Monaghan. Photo: National Museum of Ireland.

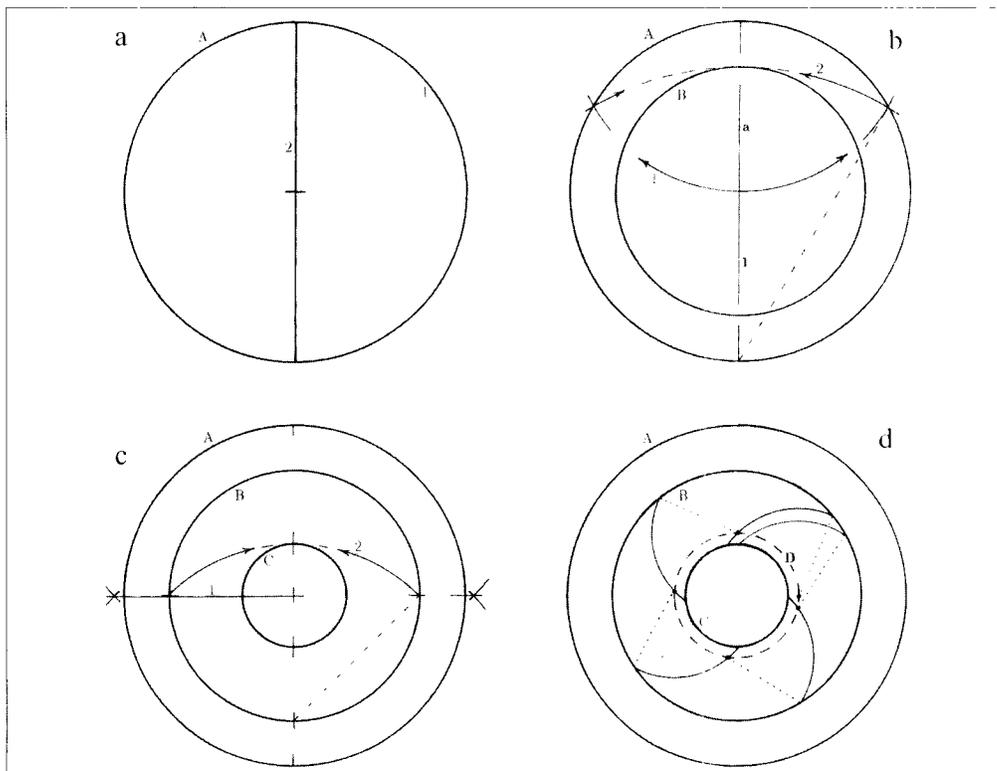


Fig. 8 A way to design the mount from Cornalaragh, Co. Monaghan.

- 8a.1 Describe a circle A .
- 8a.2 Run a straight line through the centrepoint of the circle (a diameter), dividing the circle into halves.
- 8b.1 With its original setting, set the fixed point of the compass at one end of the diameter and mark two points on the circle (cf. 6b and 10b).
- 8b.2 Copy the measure of a chord of one-third of the circle along the diameter to set the radius of circle B , concentric with circle A (same as in 6b).
- 8c.1 Set a second diameter perpendicular to the first one, dividing the circle into fourths.
- 8c.2 Copy the measure of a quadrant chord of circle B to mark a point along a diameter, and describe a new circle C concentric with circles A and B (as in 2b.1 and 4b.1).

Then proceed to fill in the area between circles B and C with a fretwork pattern. This requires finding the half-measure of circle B and the half-measure of circle A . A convenient method is to use the original compass setting for each circle in turn, place the fixed point at the end of a diameter, and mark the radial measure twice along the circumference; then mark along the radius the point where a line between these two points intersects the radius (cf. 2b and 4c).

- 8d.1 Construct, as underdrawing, a new circle D with radius half that of circle B .
- 8d.2 With the compass set at half the radius of circle A , place the fixed point of the compass at a point along circle D and describe an arc extending from circle C to circle B . Where that arc intersects circle D becomes the centre of the next arc. Repeat this procedure for drawing arcs to fill the entire area between circles B and C with a network of arcs.

The coherence of the proportioning shows clearly in Table 3, especially in the middle column (for the measure $\sqrt{3}$, see explanation accompanying Table 2).

Table 3 The Cornalaragh mount.

RADIAL MEASURE	IF a STANDS FOR $\sqrt{3}-1$	APPROXIMATION
Circle A = 1	1	1.0000
Circle B = $\sqrt{3}-1$	a	0.7321
Circle C = $(\sqrt{3}-1)(\sqrt{2}-1)$	$a(\sqrt{2}-1)$	0.3032
Circle D = $1/2(\sqrt{3}-1)$	$1/2a$	0.3660

Here is a third description of the process, perhaps something like the designer's way of instructing his assistant or apprentice.

Divide a circle into halves, thirds, and fourths.

Make a ring by drawing another circle using one-third of the first one.

Make a central disc by drawing another circle using one-fourth of the second one. Then embellish the area between the ring and the disc, using one-half the measures of the two circles of the ring. Work around a circle of half the inner part of the ring with compass set at half the outer part of the ring, drawing arcs between the ring and the disc, each new arc with a centre where the last arc crossed the half-measure of the inner circle of the ring.

Like the outer arc within the Dunadd trial-piece, the inner circle of the ring has a radius $\sqrt{3}-1$ in relation to the outer circle. In this plan that measure occurs in combination with $\sqrt{2}-1$ for the central circle in relation to the inner circle of the ring; this is the same combination that is used later in the plan of the frame and ringed cross on Soiscél Molaise, a book-shrine, for example.¹⁸

On this basic form of the Cornalaragh mount is then developed a lattice-like pattern of perforations having thirty-eight divisions. Among them are a fair number of irregularities, some of them no doubt resulting from the difficulties of chiselling at the scale of this piece (74 mm); some, however, are very likely the result of the process of the layout method, which will not produce perfect symmetry among the thirty-eight divisions of the area between ring and central disc. Among the outer openings of the lattice are four distinctive round cells. They occur in pairs along two approximately parallel lines. Their disposition is asymmetrical, nonetheless, the intervals between them being in the circular sequence 11-9-8-10. The sort-of-spiral device in the central disc is asymmetrical; despite the similarity between the large parts of the central cut-out areas, the shapes of the metal parts are far from identical. This may be outside the governing scheme—artistic improvisation—or simple error.

Somme-Bionne disc Still another example is the Somme-Bionne bronze disc, Marne (Fig. 9). It is regularly associated with the Cuperly disc both for its find-place and date, late fifth century B. C., and for its well studied art-historical affinities. Its diameter

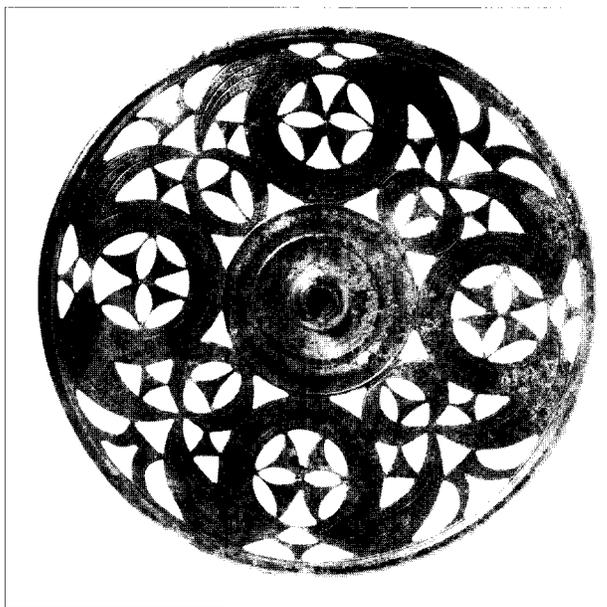


Fig. 9 Bronze openwork phalera from Somme-Bionne (Marne), B. M. ML. 1369. Photo: The British Museum.

is 6.9 cm.¹⁹ The immediately noticeable difference marking the Somme-Bionne disc is the elaborate cut-out work within and around the circles that make up the basic plan. Here is a derivation of the form of this piece (Fig. 10).

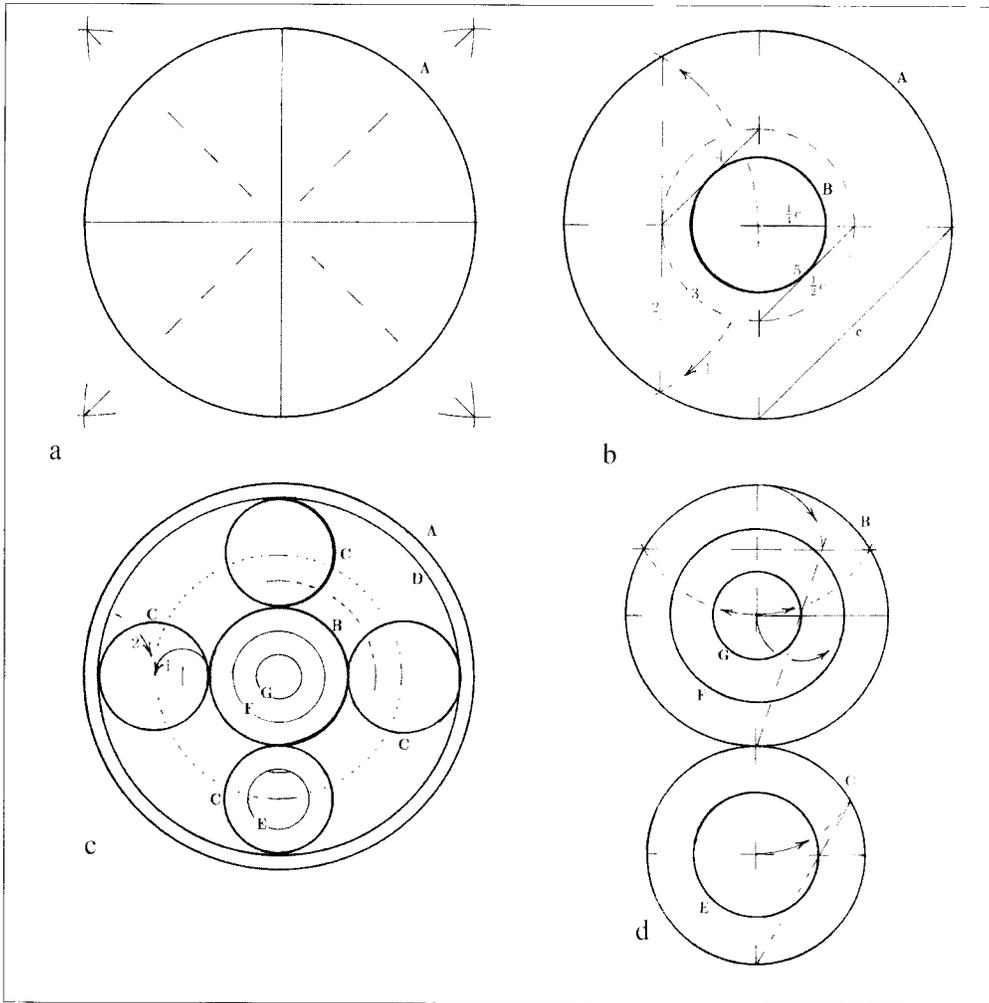


Fig. 10 A way to design the Somme-Bionne disc.

- 10a.1 Describe a circle *A*.
- 10a.2 Run a straight line—a diameter—through the centrepoint of the circle, dividing the circle into halves.
- 10a.3 Set a second diameter perpendicular to the first one, dividing the circle into quadrants.

- 10a.4 Set third and fourth diameters dividing the quadrants equally.
- 10b.1 Halve the radial measure (1, 2) (cf. 2b.3). Then sketch the path of a circle (3) around the centre of circle *A*, marking that half measure along the other axes.
- 10b.2 Sketch the chord of a quadrant of that half-measure circle (4), and draw circle *B* concentric with circle *A* and tangent to that chord (5).
- 10c.1 Copy the measure between circle *B* and the half-measure points along each axis (1) to locate the centres of circles *C*; or copy the radial measure of circle *B* inward from the outer circle *A* (2) along each axis to locate the same point: circles *C* are then drawn tangent to circle *B*.
- 10c.2 Concentric with circles *A* and *B*, draw circle *D* tangent to the outer limit of circles *C*.
- 10d.1 A simple way to plot the radius of circles *E* is shown in the figure: they are concentric with circles *C*, with radius $1/\sqrt{3}$ of the measure of circle *C*. Within the solid disc at the centre (defined by circle *B*) are two concentric circles. Their radii appear to be two-thirds and one-third the radius of circle *B*. If so,
- 10d.2 Find half the radius of circle *B* (cf. 10b.1); then as in the figure find one-third of that measure, and the double of that one, along one of the axes, to set radii of circles *F* and *G*.

Within and between the circular areas comprising the basic form are the elaborate partial circle and overlapping circle filler devices. The reconstructed scheme of the disc's composition by Frey and Schwappach provides a regularised and fully symmetrical model for the cut-out patterns.²⁰ However, there is a good deal of asymmetry and irregularity among the fourfold executions of each pattern. These factors make recovery of the coherent geometry among these elements uncertain. The basic form, on the other hand, is executed quite accurately enough that a model drawn up according to the method described, when superimposed on a photograph of the object, matches almost perfectly except for the slight misalignment of the four circles labeled *C* on the two main axes.

The geometrical coherence among the formal elements of the Somme-Bionne disc is clarified in part when the complementary relations of measures 1, 2, b, c, are set out. The chord *c* of a quadrant is $\sqrt{2}$ in relation to the overall radial measure 1 (it is the measure of 1+1 at right angles, as explained earlier); and *b* is the complement to *c* along a diameter of the overall circle. For expressing the dimensions within the basic plan, *b* can be replaced with 2-*c*; relations among the three quantities represented by 1, 2, *c* thus can describe the main plan of the disc. These numerals provide yet another three-member alphabet, as it were, in which can be written the code for this disc's basic design (circles *A*, *B*, *C*, *D*), as shown in the middle column of Table 4. Circle *E* requires an additional 'geometric' measure $1/\sqrt{3}$; as its derivation demonstrates, it is simply another eventuation from the same initial step that yielded the crucial measure 1/2 (see 10b.1, above). Circles *F* and *G* can be plotted with another subset of a geometer's methods integrated with the rest of the form, yet another eventuation from that same initial step.

Table 4 The Somme-Bionne disc.

RADIAL MEASURE	IF b STANDS FOR $2-\sqrt{2}$	APPROXIMATION
Circle A = 1	1	1.0000
Circle M = $1/2$	$1/2$	0.5000
Circle B = $1/4\sqrt{2}$	$1/2(1/\sqrt{2})$	0.3536
Circle C = $1/2b$	$1/2(2-c)$	0.2929
Circle D = (B+2C)	$1/2c+2-c$	0.9393
Circle E = $1/\sqrt{3} \times 1/2b$	$1/\sqrt{3} \times 1/2(2-c)$	0.1691
Circle F = $2/3$ circle B?	$2/3 \times 1/2c$	0.2357
Circle G = $1/3$ circle B?	$1/3 \times 1/2c$	0.1179

Noteworthy is the number of times the scheme of composition of this disc, Fig. 23 in the Frey and Schwappach study, has been reproduced, even as an offset to a photographic image of the disc on the titlepage, and again along with Plate 26, in the British Museum Guide *The Gauls: Celtic Antiquities from France*.²¹ Nonetheless, the account presented here is the first to give an 'as constructed' model of the design, and in doing so to provide a generative model and explanation of its underlying geometrical coherence.

Monasterevin disc The disc N.M.I. W.3 (Fig. 11) is an excellent demonstration-piece with which to conclude this paper. It is dated to second century A. D.; its diameter is 30.6 cm.²² Here the geometrical construction has to generate a circular piece with an 'eccentrically-placed' middle circular area, the positions for a pair of balanced spirals complementing the inner circle, the location for a set of spiral-circle-spiral devices as further complements—all with still unknown purpose—and do this with clarity and economy of the terms of its meas-

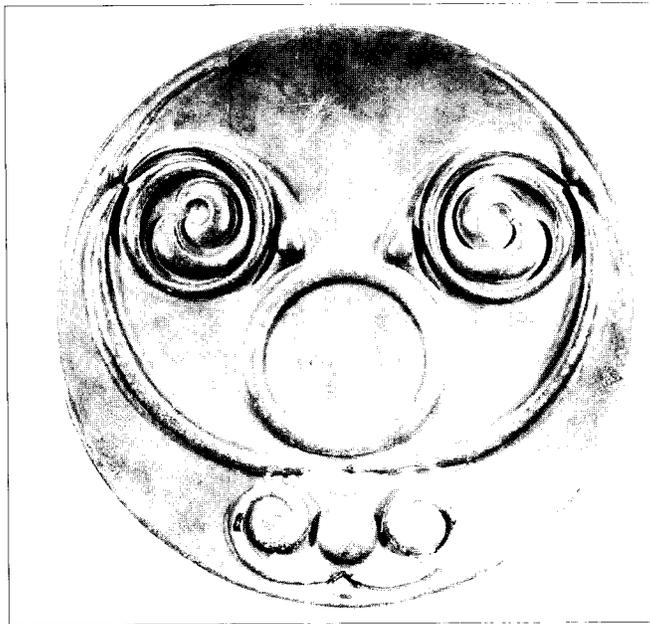


Fig. 11 Monasterevin disc, N.M.I. W.3.
Photo: National Museum of Ireland.

ures. It is one of seven known pieces of this kind, which can be described in general terms by inspection: discs 'with embossed curvilinear ornament in bronze. La Tène period'.²³ In more detail, like the others this disc is circular in shape, containing within it circular forms and some spiral pattern. None of the inner circles is concentric with the outer circle of the disc, with only the two circles forming a ring near the middle of the piece being concentric with each other. There is lateral symmetry, and there is balanced asymmetry of the upper and lower parts (in its customary orientation). A pair of smaller circular areas are laterally balanced; within them evolve opposing spirals that are extended to symmetrical curves that join below the middle circle. And so on.

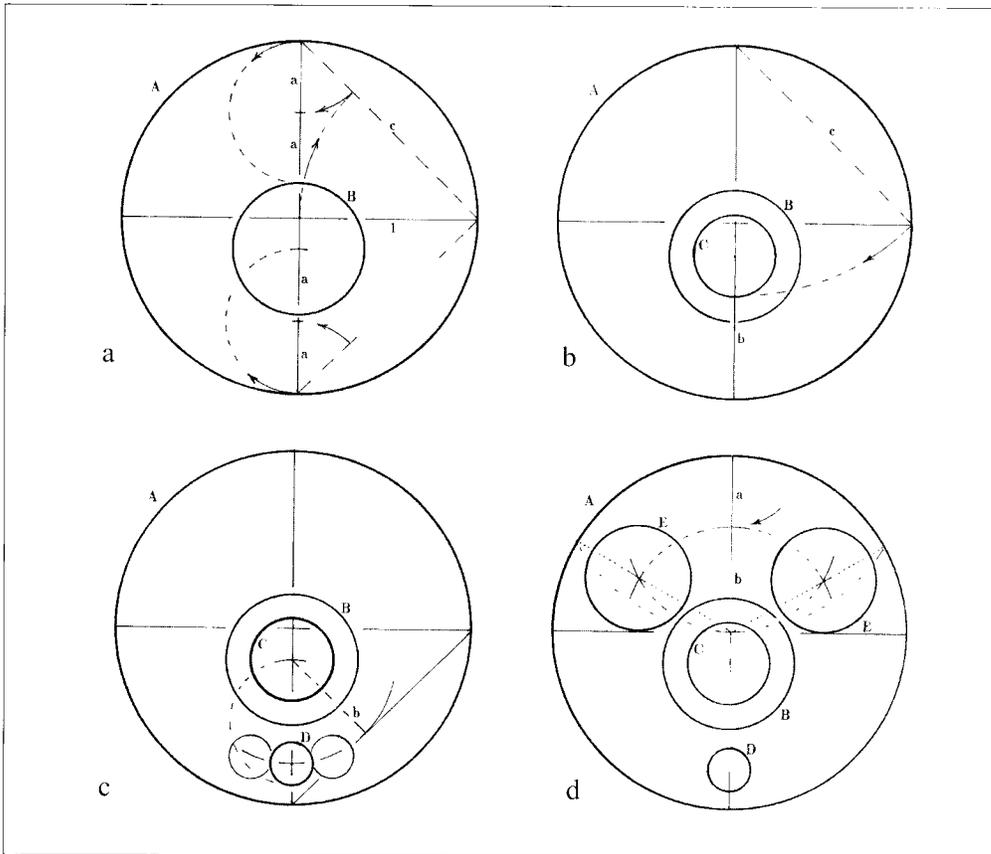


Fig. 12 A way to design Monasterevin disc, N.M.I. W.3.

As with the other pieces described in this paper, a general description can be supplemented by a list of measurements taken with a ruler. But either kind of description entails

no analysis, proposes no rationale, leaves unrecognised any method by which such a design may have been constructed exactly as it is, by plan. Hence the need once again to show how the form can be replicated (Fig. 12), together with analysis that demonstrates a commitment to total geometric coherence of its form.

The steps in a practical construction that follows are chosen for transparency and simplicity in illustrating the designing methods employed. The original designer followed a specific series of steps—with much plotting and replotting, most likely—which we cannot retrace except by speculation and guesses. The construction here provides essentially a schematic plan—what could serve as underdrawing for the initial layout. Adjustments were made later in working the design in metal, and where, for example, spirals and circle join in the disc.

As before, describe a circle with radius stipulated, and divide it into quadrants. The diameter is divided into two equal parts. As in all the other examples, the next step is to divide one of those two parts into two unequal parts. Two methods are shown. In 12a, the shorter segment *a* is located next to the circumference (and then doubled). In 12b, *a* is located next to the centre (cf. 4b.1, where the same measure was then doubled). Either, or both, can be used (and there are other methods of making the same geometrical division of the given measure). Length *a* can be expressed as $\sqrt{2}-1$, as explained earlier.

- 12a From the bottom of the plan, mark length *a* along the vertical diameter, then double it to locate the centre of circle *B*. From the top of the plan, mark length *a* along the vertical diameter, then double it to locate a point on the circumference of circle *B*. Draw circle *B*.
- 12b Copy the length of the chord (*c*) of a quadrant from the top of the plan along the vertical axis to locate a point on the circumference of circle *C*. Centre of circle *C* is the same as the centre of circle *B*. Draw circle *C*.
- 12c From the centre of circles *B/C* copy the measure *b* below along the vertical diameter to locate the centre of circle *D*. Then copy the radius of circle *B* below its lowest point—i.e., double the radius below the centre—to locate a point on the circumference of circle *D*. Draw circle *D*.
- 12d From the centre of circle *A* sketch an arc in the upper half having radius with measure *b* (complement to *a*); from a centre at *b* above the centre of circle *A*, sketch another arc also having radius with measure *b*: where these two arcs intersect will be the centres of the pair of circles *E*. Draw the pair of circles *E* tangent to the (horizontal) midline of the plan.

Here are two further observations on the construction of the design. First, the centres of the spirals on either side of circle *D* are along an arc with the same radius and centre that plots the centre of circle *D*.

Second, an alternate method for plotting circles *E* should be considered. It goes like this (shown with dotted lines in 12d). From the top of circle *A*, copy the radial length of

that circle to mark points on its circumference; this is a method of dividing the circle into thirds. Sketch radii of circle *A* to the points just marked (10 o'clock and 2 o'clock positions); where these intersect the arc concentric with circle *A* with radial measure *b* will be the centres of circles *E*. The designer may well have used both methods. This alternate procedure lays down lines useful for accurately locating the centres of the curvilinear devices just inside the perimeter of the plan.²⁴

Table 5 lists the key dimensions for the layout of this disc, with all measures expressed with only the terms 1, 2, *a*, *b*, *c*. The parsimony of plan is clearest, though, when the terms are reduced once again to three—1, 2, *c*. See 12b, where *b* appears as the complement to *c* on a diameter, hence $b=2-c$, and where *a* is the difference between *c* and 1, hence $a=c-1$.

Table 5 Monasterevin disc N.M.I. W.3.

Circle A.	Radius:	1
	Diameter:	2
Circle B.	Top:	2 <i>a</i> from top of circle <i>A</i>
	Centre:	2 <i>a</i> from bottom of circle <i>A</i>
Circle C.	Centre:	2 <i>a</i> from bottom of circle <i>A</i>
	Bottom:	<i>c</i> from top of circle <i>A</i> or <i>b</i> from bottom of circle <i>A</i>
Circle D.	Centre:	<i>b</i> below centre of circles <i>B</i> and <i>C</i> ; <i>b</i> is also the measure from centres of circles <i>B/C</i> to chord of lower quadrant of circle <i>A</i> .
	Bottom:	diameter of circle <i>B</i> below centre of circle <i>B</i>
Circle E.	Centre:	intersection of two arcs with radius <i>b</i> , one with centre at centre of circle <i>A</i> , the other with centre at <i>b</i> above centre of circle <i>A</i> (or, intersection of radius of 1/3 of circle <i>A</i> with arc having centre of circle <i>A</i> and radius <i>b</i>)
	Radius:	$\frac{1}{2}b$ (its centre to (horizontal) midline of circle <i>A</i>)

SUMMARY

These six pieces have designs which clearly are members of a single genotype. Each of them unfolds its form by the same kind of generative process, sharing in what was called at the beginning of this paper the essence of coherent geometry in Insular designing; it may be—and may always have been—esoteric, without ever having been occult. Their phenotypic differences reflect variations over time as well as adaptation to differences in locale, purpose, and material composition. The ancestry of this durable and versatile principle of designing in Insular art can thus be traced through something perhaps even more definitive than 'style' or 'motifs' to La Tène art in the early Iron Age.

NOTES

- 1 Robert D. Stevick, *The Earliest Irish and English Bookarts: Visual and Poetic Forms before A. D. 1000* (Philadelphia, 1994).
- 2 Robert D. Stevick, 'Shapes of Early Sculptured Crosses of Ireland', *Gesta*, XXXVIII (1999), 3-21; 'The Shape of the Durrow Cross', *Peritia*, 13 (1999), 142-53; 'The Coherent Geometry of Two Irish High Crosses', *Peritia*, 14 (2000), 297-322.
- 3 Robert D. Stevick, 'The Form of the Tara Brooch', *JRSAI*, 128 (1998), 5-16; 'The Form of the Hunterston Brooch', *Medieval Archaeology*, XLVII (2003), 21-39.
- 4 Rhoda Cronin, 'Late High Crosses in Munster: Tradition and Novelty in Twelfth-century Irish Art', *Early Medieval Munster: Archaeology, History and Society*, ed. Michael A. Monk and John Sheehan (Cork, 1998), 142-45.
- 5 The length of the arms and upper extension of the cross, relative to the diameter of the ring, is identical to, for example, that of both the North Cross and the South Cross, Castledermot, Co. Kildare.
- 6 The midpoint between circles *C* and *A* being equal to the sum of the radii of circles *B* and *D* can be reckoned as follows: the midpoint between circles *C* and *A* is at a distance from the centre of the plan equal to the average of the radii of circles *C* and *A*. i.e., $1/2(C+A)$, which simplifies to $1/2C+1/2A$, or $B+D$.
- 7 Françoise Henry, *L'art Irlandais*, III (1964), 175.
- 8 The tapering would be somewhat like the Killamery cross, Co. Kilkenny, or Muirdach's Cross, Co. Louth. The massiveness and consequent need for tapering of the Kilfenora cross shaft would not be necessary, of course, as is shown by the tall crosses at Moone, Co. Kildare, and at Monasterboice, Co. Louth, both of which have slender, untapered shafts.
- 9 See n. 2.
- 10 Ruth and Vincent Megaw, *Celtic Art from Its Beginnings to the Book of Kells*, 2nd ed. (2001), ill. 60, list it as 10.8 cm. I. M. Stead, *The Gauls: Celtic Antiquities from France* (c. 1981), 31, lists it as c. 111 mm. Its measure is given as 12.5 cm in Paul Jacobsthal, *Early Celtic Art* (1969), no. 179.
- 11 See Michael Ryan, 'The Early Medieval Celts', in *The Celts*, ed. Sabatino Moscati, Otto Hermann Frey, et al (1991). 'The Celtic La Tène style was established in Britain and Ireland in the prehistoric Iron Age, and appears to have survived the Roman colonization, however tenuously, in the south of Britain but more vigorously in the north and in Ireland.... [T]he style which emerges in the early medieval period, while owing its spirit to the La Tène tradition, is strongly influenced by provincial Roman types and motifs such as palmette derivatives and peltae.... Of the most common motifs which can be traced to the Iron Age tradition, two, the trumpet scroll ... and the spiral with bird-headed ... ending are the most obvious'.... 'The maturity of the new art is achieved in manuscripts in the Book of Lindisfarne ... and, in metalwork, with the "Tara" brooch" (628, 630).
- 12 Circle *F* does not appear in the disc, and is proposed here only to illustrate a method of plotting the pattern of twenty-six overlapping arcs that form the open lattice-work around the centre. It may have been used in constructing a model before the metalwork began. In any case, it illustrates a method of laying out measures that will produce the pattern that was executed in the disc itself. Circle *G*, however, is represented in the disc.
- 13 Differences between the analyses offered here for the Cuperly disc and Somme-Bionne disc (described later), and those by Majolie Lernerz-de Wilde nearly thirty years ago, *Zirkelornamentik in der Kunst der Latènezeit* (Münchner Beiträge zur Vor- und Frühgeschichte 25), München 1977, need to be noted and examined. They are two.

The first appears in the expression of relative measures in *Zirkelornamentik*, both in descriptive text and tabular listings, these measures given as simple numerals and fractions. For example, $r/2$ is one-half the overall or given radius measure; $11r/16$ is eleven-sixteenths of that measure; $3r/4$ and $7r/8$ are similar. The often-cited measures identified as the golden section of r are regularly given as $3r/8$ (shorter segment) and $5r/8$ (longer segment). The second appears in the mode of description, whether in text or in diagrams. This earlier study lists, or itemises, the measures as found and assessed.

In the present study, instead of a system employing only rational numbers, this one employs—indeed depends upon—quantities expressible only as irrational numbers: $\sqrt{2}$, $\sqrt{5}$, $\sqrt{3}$, and their combinations with 1 and 2 (from which they derive in the designs). These terms of measurement correspond to the different mode of study employed in this paper, in which the forms are analysed as products of geometrical construction. The chain of derivations of key measures, in the construction process, become a web, or network, of related dimensions in the static form of the artefact's design.

The analyses of form proposed here provide two improvements, I think, over those in *Zirkelornamentik*. One is that the newer analyses display the integration of the proportions within the plans—their 'coherent geometry', an element in both the conception and the construction of the forms. The other is that the dimensions arrived at (predicted) by the construction sequences are generally more accurate.

- 14 'Studies in Early Celtic Design,' *World Archaeology*, vol. 4 (1972), 339 ff., esp. 349 and fig. 22.
- 15 Lloyd and Jennifer Laing, *The Art of the Celts* (London, 1992), 46.
- 16 Alan Lane and Ewan Campbell, *Dunadd: An Early Dalriadic Capital*, Cardiff Studies in Archaeology (Oxford, 2000), 243-47.
- 17 Barry Raftery, *Pagan Celtic Ireland* (London, 1994), pl. 63. Also Eamonn P. Kelly, 'The Iron Age', in Patrick F. Wallace and Raghnaid Ó Floinn, eds. *Treasures of the National Museum of Ireland. Irish Antiquities* (Dublin, 2002), no. 4:24, p. 143.
- 18 Stevick (n. 1), pp. 5-8.
- 19 Ruth and Vincent Megaw (n. 10), p. 63.
- 20 'Studies in Early Celtic Design' (note 14, above), fig. 23.
- 21 I. M. Stead, *The Gauls* (see note 10). Further, the 'case studies' by Bacalt and Flouest ('Schémas des constructions des décors au compas des phalères laténiennes de Champagne', in *Décors, images et signes de l'âge du Fer européen*, ed. O. Buchsenschutz et al (2003), 145-170), focus on decoration of this disc, and claim that two concentric circles in the plan were related as 1 and $1/\phi$: 'ce cercle ne joue aucun rôle ni dans la construction préparatoire, ni dans le décor permanent: il est tellement peu important que lorsqu'on le reporte sur la photo agrandie (Stead, Rigby 1999, jaquette), il ne souligne exactement que deux jonctions de "pétales" sur les quatre' (159).
- 22 Eamonn P. Kelly (n. 17), no. 4:28, p. 144.
- 23 Barry Raftery, *La Tène in Ireland* (1984), p. 276.
- 24 In either case, it will be observed that the lower segments of the 'eye' spirals do not reach to the segments of the circles E that are tangent to a diameter. However, the circles in the plan follow the outlines of approximately the upper three-fourths of the 'eyes', and seem to be correct for both their centres and the radii.

ACKNOWLEDGEMENTS

Excellent criticism and suggestions for revision of this paper during its preparation were given generously by Professors Vincent Megaw and Thomas Elwood Hart. Further excellent criticism and advice were provided for the paper in its completed form by two referees—unnamed—who read it for *JRSAI*. The tricky and time-consuming conversion of the authors text on TEX format to HTML and thence to MSWORD (a degenerative process) was performed by Dr Stacy L. Waters.