Syntax and Alphabet

By syntax in this context is meant the rules for combining measures so that each measure is related to all others by rule, creating the coherent geometry of a design. Again and again the illustrations of coherent plans have been built from a sequence of continuous derivations by means of compass and straight-edge construction. In the designs of the best works, there are no elements of form not accountable to such a continuous, closed scheme of creation. Here is an explication of the syntax of coherent geometry, so to speak, as it appears in Insular art.

Recall that most designs are bilaterally symmetrical. For these there is an initial division of the given measure into $1 + 1$. In the designs evolved from geometrical ratios, the next step regularly is the division of one of the two equal measures into two segments that are unequal. With one the cut comes at $\sqrt{2} - 1$. With another it is at $\sqrt{3} - 1$. With the other it is at $\frac{1}{2}\sqrt{5} - 1$. (The measure 1 remains the *other* of the two parts of the given dimension.) The similarity holds for all three of these divisions except for the halving of the one with $\sqrt{5}$, that exception being necessitated by $\sqrt{5}$ being greater than 2. Just as in the creation of the musical scale—which is encompassed by two pitches related as $2:1$ (an 'octave')—the Insular designs do not exceed the ratio $2:1$ in their areas. In fact, most of them stay within a range corresponding to the musical intervals of fifth and major third.

In the unequal binary division of one of the two equal measures, there are further related ratios implicit, and that is a key to the coherence of the spatial schemes that we see in so many of the forms.

A very modern way to unfold that coherence is to express these divisions in alphanumeric terms, 1 for each of the two equal divisions, a and b for each of the unequal divisions of 1. These elements form an alphabet, as it were, in which can be 'written' the code for designs of the best and the characteristic pieces of Insular art.

With division of one half the overall measure at $\sqrt{3}-1$, there are two lengths, designated in Fig. 5.1 as a and b (cf. Figs. 3.17, 4.1b, and 4.6d). To follow this in numerical approximations, the values are

$\sqrt{3} = 1.73205 \qquad a = 0.73205 \qquad b = 0.26795$

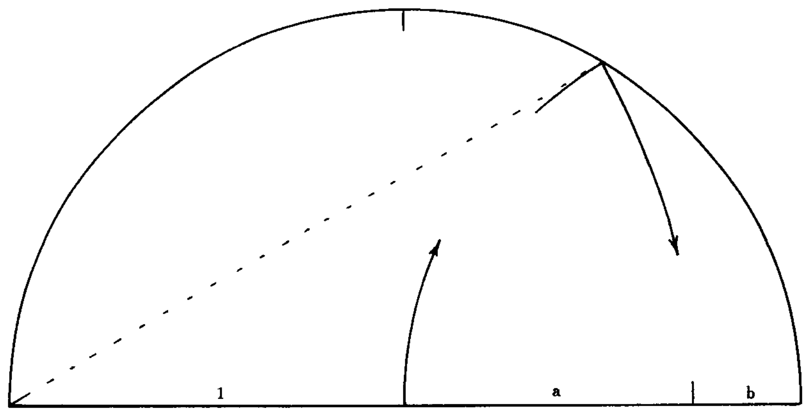

Fig. 5.1 Dividing half of a given measure at $\sqrt{3}-1$.

Here are some of the proportional linkings of 1, 2, a, and b:

$1+1=2$

$a+b=1$

$b/a = a/2 \quad = 1/(a+2) \quad = \frac{1-a}{1-b} \quad = \frac{1-a}{a}$

$\frac{1+a}{a} = 2 + \frac{a}{2}$

These are the most immediate combinations for equal ratios, and all that are employed in nearly all the spatial plans built around $\sqrt{3}$ geometry.

The tautology among the various formulations will be obvious in their algebraic expression above. It becomes radical when the first two equivalences are considered: $1+1=2$ and $a+b=1$. These of course correspond to the initial dividing of the given measure equally by 2, then dividing one of the halves by using the geometrical measure $\sqrt{3}$. It will be further obvious, then, that all the relations can be reduced to 1 and a as their only terms, as is the case in several of the formulations above.

With division of one half the overall measure at $\sqrt{2}-1$, there are two lengths, designated in Fig. 5.2 as a and b (cf. Fig. 3.11c). To follow this in numerical approximations, the values are

$\sqrt{2} = 1.4142 \qquad a = 0.4142 \qquad b = 0.5858$

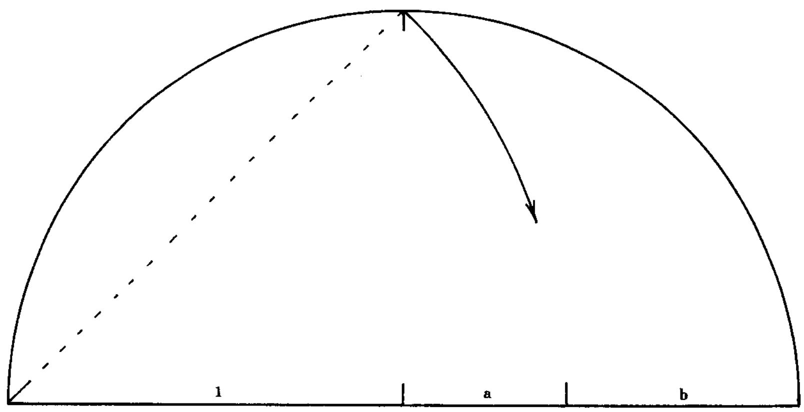

Fig. 5.2 Dividing half of a given measure at $\sqrt{2}-1$.

Here are some of the proportional linkings of 1, 2, a, and b:

$1 + 1 = 2$

$a + b = 1$

$b/a = 1 + a \quad$ hence $\frac{1+a}{b} = 2 + a$

$1/a = 1 + (1 + a)$

$a = \frac{1}{2+a}$

$\frac{a}{1-a} = \frac{1+a}{2}$

$\frac{1}{1+a} = \frac{1+a}{2}$, $\sqrt{2}$ (i.e., $1 + a$) being the geometric mean of 1 and 2.

These are the most immediate combinations for equal ratios, and all that are employed in many of the spatial plans built around $\sqrt{2}$ geometry. There are further divisions, such as $a/2$ and $b/2$, that enter into the schemes of several forms, as well. They merely extend the set of measures that participate in linked ratios for the coherence of forms.

The tautology among the various formulations will be obvious once more in their algebraic expression, and again it becomes radical when the first two equivalences are considered: $1 + 1 = 2$ and $a + b = 1$. And once again, all the relations can be reduced to 1 and a as their only terms.

With division of one half the overall measure at $\frac{1}{2}\sqrt{5-1}$, there are two lengths, designated in Fig. 5.3 as a and b (cf. Fig. 3.20). To follow this in numerical approximations, the values are

$\varphi = \frac{2}{\sqrt{5}-1} = 1.61803 \qquad a = 0.61803 \qquad b = 0.38197$

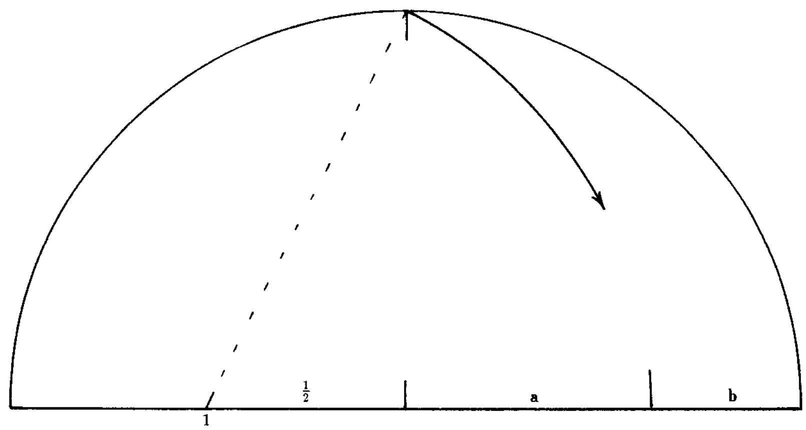

Fig. 5.3 Dividing half of a given measure at $\frac{\sqrt{5-1}}{2}$.

Here are some of the proportional linkings of 1, 2, a, and b:

$1 + 1 = 2$

$a + b = 1$

$\frac{b}{a} = \frac{a}{b+a} = \frac{1}{1+a}$

$a/b = 1/a = 1 + a$

With this geometrical division into a and b there are further measures entailed that link in a chain of equivalent ratios, forming a continuous proportion—something like 'interlace':

$$\varphi = 1 + a = \frac{1}{a} = \frac{a}{b} = \frac{b}{c} = \frac{c}{d} \ldots.$$

More of the linking appears in this display:

$$a = \frac{1}{\varphi} \qquad b = \frac{1}{\varphi^2} \qquad c = \frac{1}{\varphi^3} \qquad d = \frac{1}{\varphi^4} \ldots.$$

Like the others, the whole set can be reduced to combinations of 1 and a:

$a = \varphi - 1$

$b = 1 - a$

$c = a - b$, thus $= a - (1 - a)$

$d = b - c$, thus $= (1 - a) - (a - (1 - a))$

$e = c - d$, thus $= (a - (1 - a)) - ((1 - a) - (a - (1 - a)))$

....

With each of these geometrical sets based on $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, we have in effect the 'alphabet' with which can be written the code for the various coherent forms. In each alphabet the values of a and b (and any further derivations from them) are different, of course.

The purpose of the modern notation for the measures and the relations among them is to help in grasping and appreciating the radical coherence in the geometry of many of the designs. The 'alphabet' for writing the code of these forms can be reduced to only two members, 1 and a, an ultimately simple, binary set. The syntax requires only + and − and simple nesting (the parentheses).

When all this equivalencing is done, however, in terms quite alien to the persons who created the designs of crosses, carpet pages, poems, and some metalwork, it is most important to return to the designs themselves and the practical methods of those who created them. In craftsmen's terms, the practical alphabet of Insular design was not the stark 1 and a that we can reduce it to, but always included 2 and b and especially in the instance of the golden ratio it included c and sometimes d and e as well. Depending on where, say, measure d may be called for, it can be created or it can be copied from anywhere the appropriate equivalent may lie in the design; for example:

$$d = 2a - b$$
$$d = b - c$$
$$d = 2b - a$$
$$d = 1 - (a + c)$$
$$d = a - 2c$$

The interrelations of these measures is indeed a rich resource for designs with coherent geometry. Whether the 'alphabet' is the reduced set 1 and a or whether it includes 2 and b and as needed c, d, e ..., the syntax remains the same. It works, of course, in the two dimensions of a plane, not just the one (lineal) dimension of the algebraic notation used above.